Instabilities in Taylor-Couette-Poiseuille flow with porous walls

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INSTABILITIES IN TAYLOR–COUETTE–POISEUILLE FLOW WITH POROUS WALLS

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We consider Taylor–Couette–Poiseuille flow developing between an outer, fixed, impermeable cylinder and a concentric, inner, rotating, permeable cylinder with radial suction, see figure 1. This system is useful for

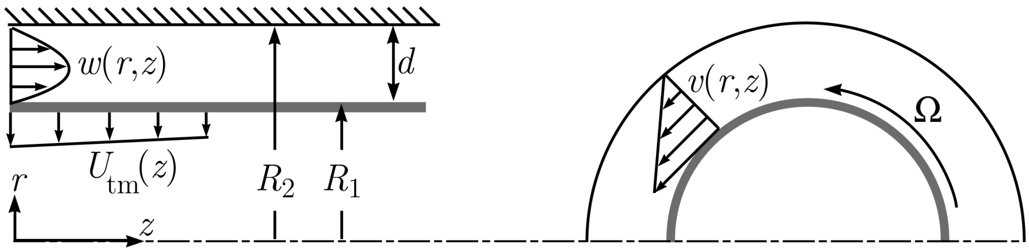

Figure 1: Sketch, not to scale, of the annular geometry and laminar base flow.

dynamic filtration because the shear due to the rotating cylinder and, in the case of supercritical flow, the Taylor vortices, wash contaminants away from the permeable cylinder and prevent fouling. The fluid mechanics of this system are not fully understood due to the coupling between the axial pressure drop, which drives the axial Poiseuille flow, and the transmembrane pressure difference which drives the suction. In addition to the filtration flux, this coupling induces axial variations of the velocity field. These variations eventually modify the nature of the subcritical flow which can evolve from suction to injection (cross flow reversal) or consume the whole axial flow (axial flow exhaustion). Moreover, owing to the axial and radial flows and their variations along the axial direction, the stability of this flow strongly departs from that of Taylor–Couette flow.

1 Analytical approach

Because filtrating devices utilize membranes with small permeability and suction, we propose an asymptotic solution to the subcritical flow assuming a slow axial variation of the velocity and pressure fields [1]. The transmembrane suction is coupled with the pressure through Darcy's law. The obtained analytical approximation correctly captures the axial variations of the velocity field, e.g. the two aforementioned possible behaviors. This laminar flow is then used as a base state to study the appearance of centrifugal instabilities in the form of Taylor vortices developing in the most common configuration, i. e. when the mean axial flow decreases downstream due to filtration but do not reverse. According to the theory of nonlinear global modes in slowly varying open flows [2], the unstable state is expected to form a front at the axial location where the flow undergoes a transition from local convective unstability to local absolute instability. This front acts as a wavemaker and selects the frequency of the vortices.

2 Numerical approach

These analytical results for the subcritical and supercritical flows are then compared with dedicated spectral direct numerical simulations implementing Darcy's law on the inner cylinder. These numerical simulations require some special care to correctly handle the axial and transmembrane flows, together with solving the pressure field, in order to avoid numerical noises prone to trigger uncontroled extrinsic instabilities.

3 Results

As seen in figure 2, global synchronized modes governed by local absolute instabilities are retrieved in numerical simulations. These numerical results, concerning for instance the features of these instabilities such as the location of the front or the frequency, are in good agreement with the analytical prediction based on previous

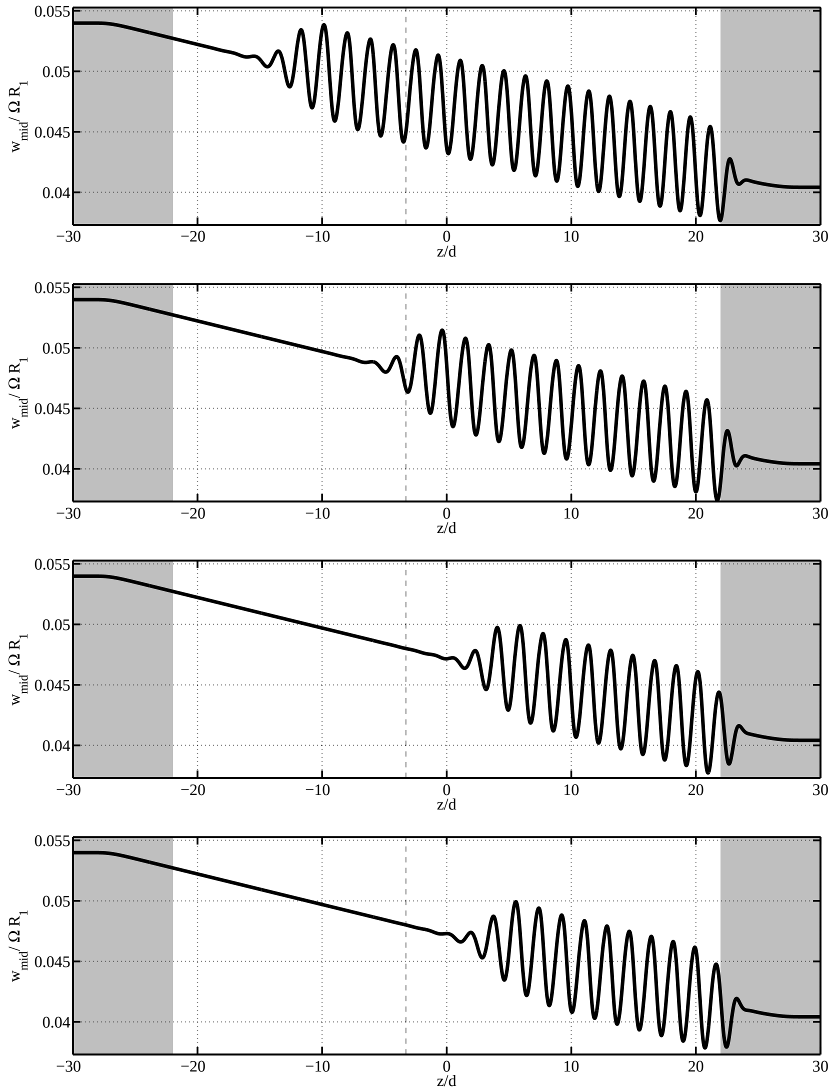

Figure 2: Temporal evolution of the axial velocity as a function of the axial coordinate, the rotating Reynolds number is set to 120, the axial Reynolds number varies from 4.32 (inlet) to 3.24 (outlet). (−−): Boundary between the locally convectively and absolutely unstable regions

results pertaining to the convective/absolute stability analysis of the Taylor–Couette–Poiseuille without radial flow and the axial evolution of the stationnary part of the total flow.

References

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